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Fuzzy Optimization Technique for Pareto Optimal Solution of Structural Models with Stress constraints

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Abstract: Multi-objective non-linear programs occur in various field of engineering application. One of the applications of such program is structural design problem. In this paper, we consider a generalized form of a multi-objective structural design problem. Triangular norm based fuzzy programming technique is used to solve these problem. The test problem includes a three-bar planar truss subjected to a single load condition. The model is illustrated with numerical examples.

Keywords: Structural design, Fuzzy sets, Pareto optimal solution model, Fuzzy optimization technique, Multi-objective programming, t-norm.

I. INTRODUCTION

Optimization is the process of minimizing or maximizing an objective function (e.g. cost, weight) of a structural system which has been frequently employed as the evaluation criterion in structural engineering applications. But in the practical optimization problems, usually more than one objective are required to be optimized, such as minimum mass or cost, maximum stiffness, minimum displacement at specific structural points, maximum natural frequency of free vibration, and maximum structural strain energy. This makes it necessary to formulate a multi-objective optimization problem. The first note on multi-objective optimization was given by Pareto; since then the determination of the compromise set of a multi-objective problem is called Pareto optimization. That is why the application of different optimization technique [11,16-19] to structural problems has attracted the interest of many researchers.

In conventional mathematical programming, the coefficient or parameters of mathematical models are assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may be somewhat uncertain in nature. Thus the decision making methods under uncertainty are needed. The fuzzy programming has been proposed from this point view. In decision making process, first Zadeh[2] introduced fuzzy set theory. Tanaka et al.[20]applied the concept of fuzzy sets to decision making problems by considering the objectives as fuzzy goals. Later on Bellman and Zadeh [3] used the fuzzy set theory to the decision making problem. Zimmermann [4] proposed a fuzzy multi-criteria decision making set, defined as the intersection of all fuzzy goals and their constraints.

In practical, the problem of structural design may be formed as a typical non-linear programming problem with non-linear objective functions and constraints functions in fuzzy environment. Some researchers applied the fuzzy set theory to Structural model. For example Wang et al. [1] first applied α -cut method to structural designs where the non-linear problems were solved with various design levels α , and then a sequence of solutions are obtained by setting different level-cut value of α . Rao [8] applied the same α -cut method to design a four–bar mechanism for function generating problem .Structural optimization with fuzzy parameters was developed by Yeh et.al [7]. In 1989, Xu [6] used two-phase method for fuzzy optimization of structures. In 2004, Shih et.al [9] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources .Shih et.al [10] develop an alternative α -level-cuts methods for optimum structural design with fuzzy resources in 2003. Dey et.at [5] optimize structural model in fuzzy environment.

Alsina et.al. [13] introduced the t-norm into fuzzy set theory and suggested that the t-norms be used for the intersection of fuzzy sets. Different types of t-norms theory and their fuzzy inference methods were introduced by Gupta et.al.[14] .The extension of fuzzy implication operators and generalized fuzzy methods of cases were discussed by Ruan et.al. [15].

In this paper we propose an approach to solve multi-objective structural model using t-norms based fuzzy optimization programming technique. In this structural model formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. The test problem includes a three-bar planar truss subjected to a single load condition. This approximation approach is used to solve this multi-objective structural optimization model.

The remainder of this paper is organized in the following way. In section II, we discuss about structural optimization model. We discuss about mathematics Prerequisites and aggregation operator in section III and IV respectively. In section V, we discuss fuzzy optimization technique to solve multi-objective non-linear programming problem. In section VI, we discuss Pareto optimality test. In section VII, we solve multi-objective structural model using t-norms based fuzzy optimization. In section VIII, numerical solution of structural model of three bar truss. Finally we draw conclusions in section IX.

II. MULTI-OBJECTIVE STRUCTURAL MODEL

In the design of optimal structure i.e. lightest weight of the structure and minimum deflection of loaded joint that satisfies all stress constraints in members of the structure. To bar truss structure system the basic parameters (including the elastic modulus, material density, the maximum allowable stress, etc.) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight, the minimum nodes displacement, in a given load conditions.

The multi-objective Structural model can be expressed as:

$$\begin{aligned}
&\textit{Minimize } WT(A)\\
&\textit{minimize } \delta(A)\\
&\textit{subject to } \sigma(A) \le [\sigma_0]\\
&\qquad A_{\min} \le A \le A_{\max}
\end{aligned} \tag{1}$$

where $A = [A_1, A_2, ..., A_n]^T$ are design variables for the cross section, n is the group number of design variables for the cross section bar, $WT = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure, $\delta(A)$ is the deflection of loaded joint L_i, A_i and ρ_i were the bar length, cross section area, and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma_0]$ is maximum allowable stress of the group bars under various conditions, $A_{\min}$ and $A_{\max}$ are the minimum and maximum cross section area respectively.

III. PREREQUISITE MATHEMATICS

III(a). Fuzzy Set:

Let X is a set (space), with a generic element of X denoted by x, that is $X(x)$. Then a Fuzzy set (FS) is defined as $\tilde{A} = \{(x, \mu_A(x)) : x \in X\}$

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of FS $\tilde{A}$. $\mu_{\tilde{A}}(x)$ is the degree of membership of the element x to the set $\tilde{A}$.

III(b). α -Level Set or α -cut of a Fuzzy Set:

The α -level set of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values greater than or equal to α i.e. $\tilde{A} = \{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

III(c). Convex fuzzy set:

A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X, $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda x_2) \ge \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)))$ when $0 \le \lambda \le 1$.

IV. AGGREGATION OPERATOR

When the rules in the decision support system contain more than one antecedent, the degrees of strength of antecedents need to be combined to determine the overall strength of the rule consequent. In the language of fuzzy sets, the membership values of the linguistic variables in the rule antecedents have to be combined using an aggregation operator. Formally, a general aggregation is a real function $T : [0,1]^n \to [0,1]$, non decreasing in all arguments, with the properties $T(0) = 0$ and $T(1) = 1$.

General aggregation operators display the whole range of behavior, disjunctive, conjunctive, averaging, mixed, commutative, mutually reinforcing or otherwise and correspond to vague and loosely defined "and" and "or" connectives etc. Triangular norms and conforms and averaging operators are well known examples of the

aggregation operators. Different class of aggregation operators display substantially different behavior, it is not logical to use any particular class to provide generic representation of aggregation. Therefore, we will use general aggregation operators to model aggregation of rule antecedents in decision support systems. They will provide the highest degree of adaptability and excellent empirical fit. However, if there are strong reasons to restrict the selection to a particular family of operators, we will impose the relevant constraints.
Consider general aggregation operator. The function can have a simple algebraic form, such as

$T(x_1,x_2,....,x_n)=\min\{x_1,x_2,......x_n\}$

or $T(x_1,x_2,....,x_n)=x_1x_2.......x_n=\prod_{i=1}^{n}x_i$

or $T(x_1,x_2,....,x_n)=\min\left\{1,\sum_{i=1}^{n}x_i\right\}$

or $T(x_1,x_2,....,x_n)=\dfrac{\sum_{i=1}^{n}x_i}{n}$

The degree of importance of rule antecedents (vector a) can be easily incorporated into aggregation operators in variety of ways. For example

$T(x_1,a_1;x_2,a_2;.......;x_n,a_n)=\min\{x_1,a_1\}\times\min\{x_2,a_2\}\times......\times\min\{x_n,a_n\}$

$T(x_1,a_1;x_2,a_2;.......;x_n,a_n)=\min\{x_1a_1+x_2a_2+.......+x_na_n,1\}$

In this article, decision making method used by the weighted bounded sum operator (member of Yager family of triangular conorms).

V. FUZZY PROGRAMMING TECHNIQUE TO SOLVE MULTI-OBJECTIVE NON-LINEAR PROGRAMMING PROBLEM (MONLP)

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$Minimize\ \ f(x)=[f_1(x),f_2(x),.........f_k(x)]^T \tag{2}$$

subject to $x\in X=\left\{x\in R^n : g_j(x)\le or = or \ge b_j \ \ for \ \ j=1,2,3,....,m\right\}$ and $l_i\le x_i\le u_i \quad (i=1,2,3,....,n)$

Zimmermann (1978) showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

Step 1: Solve the MONLP (2) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2: From the result of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ .... \\ x^k \end{array}\begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & .... & f_k(x) \end{array} \\ \begin{pmatrix} f_1^*(x^1) & f_2^*(x^1) & .... & f_k^*(x^1) \\ f_1^*(x^2) & f_2^*(x^2) & .... & f_k^*(x^2) \\ .... & .... & .... & .... \\ f_1^*(x^k) & f_2^*(x^k) & & f_k^*(x^k) \end{pmatrix}\end{array}$$

Here x^1, x^2, x^3,….., x^k are the ideal solutions of the objectives $f_1(x)$, $f_2(x)$,….., $f_k(x)$ respectively.

So $U_r=\max\left\{f_r(x^1),f_r(x^2),......,f_r(x^k)\right\}$ and $L_r=\min\left\{f_r(x^1),f_r(x^2),......,f_r(x^k)\right\}$

Where U_r and L_r be upper and lower bounds of the r^{th} objective function $f_r(x)$ for $r=1,2,3,........,k$.

Step 3: Using aspiration level of each objective of the MONLP (3) may be written as follows:

Find x so as to satisfy

$f_r(x)\tilde{\le}L_r$ with tolerance $P_r\ (=U_r-L_r)$ for $r=1,2,3,........,k$

$x\in X$. $l_i\le x_i\le u_i \quad (i=1,2,3,....,n)$

Here objective functions of (2) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$$\mu_r(f_r(x)) = \begin{cases} 0 & if \;\; f_r(x) \geq U_r \\ \dfrac{U_r - f_r(x)}{U_r - L_r^{'}} & if \;\; L_r^{'} \leq f_r(x) \leq U_r \\ 1 & if \;\; f_r(x) \leq L_r^{'} \end{cases}, \quad (3)$$

where $L_r^{'} = L_r + \in_r$ and $0 \leq \in_r \leq U_r - L_r$, for $r = 1, 2, 3, ..., k$

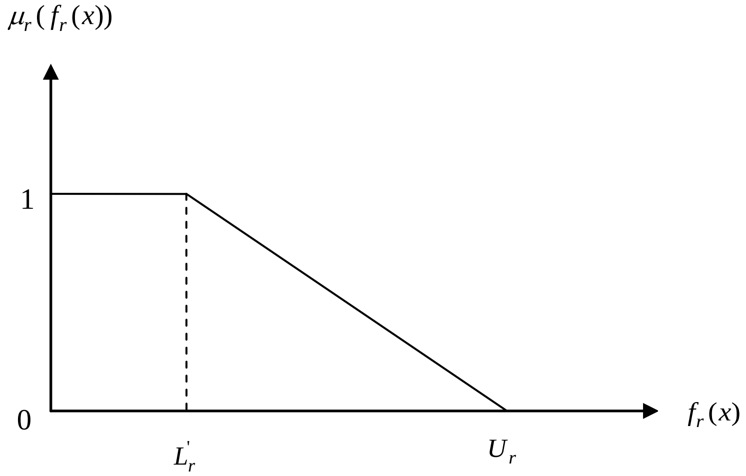

Figure 1: Membership function for objective functions $f_r(x)$

Having elicited the membership functions as in (3) $\mu_r(f_r(x))$ for $r = 1, 2, 3, ..., k$ a general aggregation function $\mu_{\tilde{D}}(x) = F(\mu_1(f_1(x)), \mu_2(f_2(x)), ..., \mu_k(f_k(x)))$ is introduced.

So a fuzzy multi-objective decision making problem can be defined as

$$\begin{aligned} &Maximize \;\; \mu_{\tilde{D}}(x) \\ &subject \; to \;\; x \in X. \end{aligned} \quad (4)$$

Fuzzy decision making method used by the (weighted) bounded sum operator (member of Yager family of triangular conforms) the problem (4) is reduced to

$$\begin{aligned} &Maximize \;\; \mu_{\tilde{D}}(x; W) = \sum_{r=1}^{k} W_r \mu_r(f_r(x)) \\ &subject \; to \; x \in X \\ &0 \leq \mu_r(f_r(x)) \leq 1 \quad for \;\; r = 1, 2, .., k \\ &where \;\; W_r \geq 0 \quad for \; all \; r = 1, 2, ..., k, \; \sum_{r=1}^{k} W_r = 1 \end{aligned} \quad (5)$$

Step 4: Solve (5) to get optimal solution.

Some basic definitions are introduced below.

V(a). Complete Optimal Solution

x^* is said to be a complete optimal solution to the MONLP (3) if and only if there exists $x \in X$ such that $f_r(x^*) \leq f_r(x)$ for $r = 1, 2, ..., k$ and for all $x \in X$ .However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and hence the Pareto Optimality Concept arises and it is defined as follows.

V(b). Pareto Optimal Solution

x^* is said to be a Pareto optimal solution to the MONLP (3) if and only if there does not exist another $x \in X$ such that $f_r\left(x^*\right) \le f_r\left(x\right)$ for all $r = 1,2,...,k$ and $f_j\left(x\right) \ne f_j\left(x^*\right)$ for at least one j, $j \in \{1,2,...,k\}$.

VI. PARETO OPTIMALITY TEST

A numerical test of Pareto optimality for x^* can be performed by solving the following problem:

$$Maximize\ R = \sum_{r=1}^{k} \varepsilon_r$$

$$subject\ to\ f_r\left(x\right) + \varepsilon_r = f_r\left(x^*\right), r = 1,2,...,k \tag{6}$$

$$x \in X;\quad \varepsilon_r \ge 0.$$

The optimal solution of (6), say x^{**} and $f_r\left(x^{**}\right)$ are called strong Pareto optimal solution provided V is very small otherwise it is called weak Pareto solution.

VII. FUZZY PROGRAMMING TECHNIQUE IN MULTI-OBJECTIVE STRUCTURAL MODEL

To solve the above MOSOP (1), step 1 of V is used. After that according to step 2 pay-off matrix formulated as follows:

$$\begin{array}{cc} & \begin{array}{cc} WT(A) & \delta(A) \end{array} \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{pmatrix} WT^*(A^{1*}) & \delta(A^{1*}) \\ WT(A^{2*}) & \delta^*(A^{2*}) \end{pmatrix} \end{array}$$

After that according to step 2, the bounds of objective are U_1, L_1 for weight function $WT(A)$ (where $L_1 \le WT(A) \le U_1$) and the bounds of objective are U_2, L_2 for deflection function $\delta\left(A\right)$ (where $L_2 \le \delta\left(A\right) \le U_2$) are identified.

Above MOSOPP reduces to a FMOSOPP as follows;

Find A

Such that

$WT\left(A\right) \tilde{\le} L_1$ with maximum allowable tolerance $P_1\left(= U_1 - L_1\right)$

$\delta\left(A\right) \tilde{\le} L_2$ with maximum allowable tolerance $P_2\left(= U_2 - L_2\right)$

$$\sigma(A) \le [\sigma_0]$$

$$A_{\min} \le A \le A_{\max}$$

Here for simplicity linear membership functions $\mu_{WT}\left(WT\left(A\right)\right)$ and $\mu_\delta\left(\delta\left(A\right)\right)$ for the objective functions $WT\left(A\right)$ and $\delta\left(A\right)$ respectively are defined as follows:

$$\mu_{WT}\left(WT\left(A\right)\right) = \begin{cases} 1 & if\ WT\left(A\right) \le L_1^{'} \\ \left(\dfrac{U_1 - WT\left(A\right)}{U_1 - L_1^{'}}\right) & if\ L_1^{'} \le WT\left(A\right) \le U_1 \\ 0 & if\ WT\left(A\right) \ge U_1 \end{cases}$$

$$\mu_\delta\left(\delta\left(A\right)\right) = \begin{cases} 1 & if\ \delta\left(A\right) \le L_2^{'} \\ \left(\dfrac{U_2 - \delta\left(A\right)}{U_2 - L_2^{'}}\right) & if\ L_2^{'} \le \delta\left(A\right) \le U_2 \\ 0 & if\ \delta\left(A\right) \ge U_2 \end{cases}$$

According to step-3, having elicited the above membership functions crisp non-linear programming problem is formulated as follows

$$Maximize\ \left(W_1\mu_{WT}\left(WT\left(A\right)\right)+W_2\mu_{\delta}\left(\delta\left(A\right)\right)\right) \tag{7}$$

$$subject\ \ to$$

$$\begin{gathered} 0\le \mu_{WT}\left(WT\left(A\right)\right)\le 1,\ \ 0\le \mu_{\delta}\left(\delta\left(A\right)\right)\le 1, \\ \sigma(A)\le[\sigma], \\ A_{\min}\le A\le A_{\max}, \\ W_1\ge 0, W_2\ge 0,\ W_1+W_2=1; \end{gathered}$$

The problem (7) can be written as

$$maximize\ \left(W_1\left(\frac{U_1-WT\left(A\right)}{U_1-L_1^{'}}\right)+W_2\left(\frac{U_2-\delta\left(A\right)}{U_2-L_2^{'}}\right)\right) \tag{8}$$

$$subject\ \ to$$

$$\begin{gathered} 0\le\left(\frac{U_1-WT\left(A\right)}{U_1-L_1^{'}}\right)\le 1,\ \ 0\le\left(\frac{U_2-\delta\left(A\right)}{U_2-L_2^{'}}\right)\le 1, \\ \sigma(A)\le[\sigma], \\ A_{\min}\le A\le A_{\max}, \\ W_1\ge 0, W_2\ge 0,\ W_1+W_2=1; \end{gathered}$$

VIII. NUMERICAL SOLUTION OF A MULTI-OBJECTIVE STRUCTURAL OPTIMIZATION MODEL OF A THREE-BAR TRUSS

A well-known three bar [12] planar truss structure is considered. The design objective is to minimize weight of the structural $WT\left(A_1,A_2\right)$ and minimize the deflection $\delta\left(A_1,A_2\right)$ along x and y axes at loading point of a statistically loaded three-bar planar truss subjected to stress $\left(\sigma\right)$ constraints on each of the truss members.

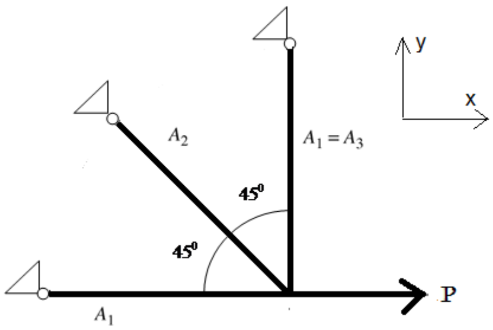

Figure 2. Design of the three-bar planar truss

The multi-objective optimization problem can be stated as follows:

$$
\begin{aligned}
&\text{Minimize } WT(A_1,A_2)=\rho L(2A_1+A_2)\\
&\text{minimize } \delta_x(A_1,A_2)=\frac{PL(2A_1+A_2)}{E(2A_1^2+2A_1A_2)}\\
&\text{minimize } \delta_y(A_1,A_2)=\frac{PLA_2}{E(2A_1^2+2A_1A_2)}\\
&\text{subject to } \sigma_1(A_1,A_2)\equiv\frac{P(2A_1+A_2)}{2A_1A_2+2A_1^2}\le\left[\sigma_1^T\right]\\
&\qquad\sigma_2(A_1,A_2)\equiv\frac{P}{\sqrt{2}(A_1+A_2)}\le\left[\sigma_2^T\right]\\
&\qquad\sigma_3(A_1,A_2)\equiv\frac{PA_2}{2A_1A_2+2A_1^2}\le\left[\sigma_3^C\right]\\
&\qquad\qquad A_i^{\min}\le A_i\le A_i^{\max};\ i=1,2
\end{aligned}
\tag{9}
$$

Where P = applied load; ρ =material density, L =Length, E=Young's modulus, A_1 = cross section of bar-1 and bar-3, A_2 =cross section of bar-2. δ_x and δ_y are the deflection of loaded joint along x and y axes respectively. $\left[\sigma_1^T\right]$ and $\left[\sigma_2^T\right]$ the maximum allowable tensile stress for bar 1 and bar 2 respectively. $\left[\sigma_3^C\right]$ is the maximum allowable compressive stress for bar 3.

The input data for MOSOP (9) is given as follows

Table 1: Input data for crisp model (9)

Applied load P (KN)	Volume density ρ (KN/m^3)	Length L (m)	Maximum allowable tensile stress $\left[\sigma_1^T\right]$ (KN/m^2)	Maximum allowable tensile stress $\left[\sigma_2^T\right]$ (KN/m^2)	Maximum allowable compressive stress $\left[\sigma_3^C\right]$ (KN/m^2)	Young's modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $(10^{-4}m^2)$
20	100	1	20	10	20	2×10^8	$A_1^{\min}=0.1$ $A_1^{\max}=5$ $A_2^{\min}=0.1$ $A_2^{\max}=5$

Solution: According to step 2 pay off matrix is formulated as follows:

$$
\begin{array}{c} \\ A^1\\ A^2\\ A^3 \end{array}
\begin{array}{c}
\begin{array}{ccc} WT(A_1,A_2) & \delta_x(A_1,A_2) & \delta_y(A_1,A_2) \end{array}\\
\begin{pmatrix} 2.187673 & 20 & 5.857864\\ 15 & 3 & 1\\ 10.1 & 3.960784 & 0.039216 \end{pmatrix}
\end{array}
$$

Here $U_{WT}=15$, $L_{WT}=2.187673$, $L_{WT}^{'}=3$ $U_{\delta_x}=20$, $L_{\delta_x}=3$, $L_{\delta_x}^{'}=3.5$, $U_{\delta_y}=5.857864$, $L_{\delta_y}=0.039216$, $L_{\delta_y}^{'}=0.15$; Here linear membership for the objective functions $WT(A_1,A_2)$, $\delta_x(A_1,A_2)$ and $\delta_y(A_1,A_2)$ are defined as follows:

$$\mu_{WT}(WT(A_1,A_2)) = \begin{cases} 1 & if\ \ WT(A_1,A_2) \le 3 \\ \dfrac{15 - WT(A_1,A_2)}{12} & if\ 3 \le WT(A_1,A_2) \le 15 \\ 0 & if\ \ WT(A_1,A_2) \ge 15 \end{cases}$$

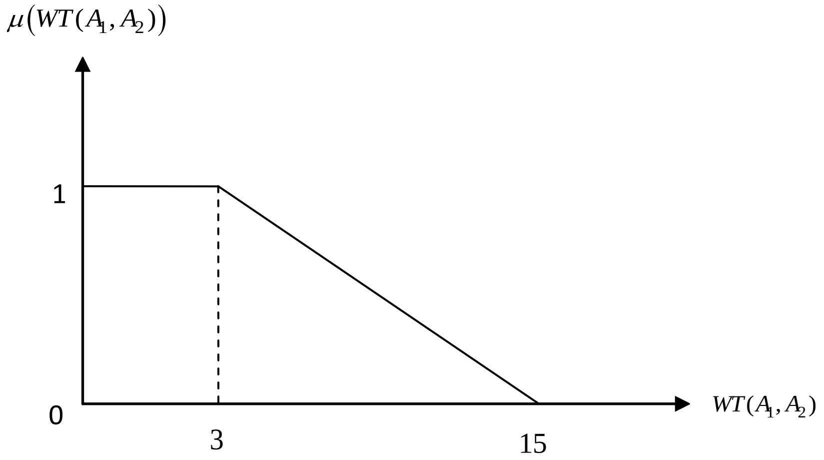

Figure 3. Membership and non-membership function for $WT(A_1,A_2)$

$$\mu_{\delta_x}(\delta_x(A_1,A_2)) = \begin{cases} 1 & if\ \ \delta_x(A_1,A_2) \le 3.5 \\ \dfrac{20 - \delta_x(A_1,A_2)}{16.5} & if\ 3.5 \le \delta_x(A_1,A_2) \le 20 \\ 0 & if\ \ \delta_x(A_1,A_2) \ge 20 \end{cases}$$

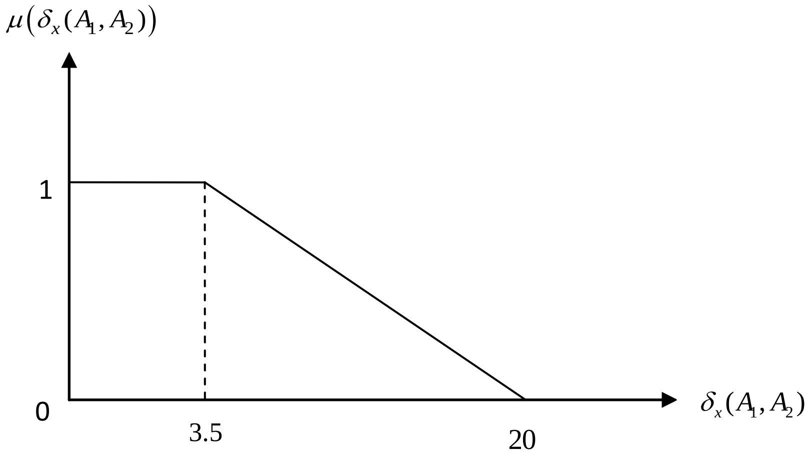

Figure 4. Membership and non-membership function for $\delta_x(A_1,A_2)$

$$\mu_{\delta_y}(\delta_y(A_1,A_2)) = \begin{cases} 1 & if\ \ \delta_y(A_1,A_2) \le 0.14 \\ \dfrac{5.857864 - \delta_y(A_1,A_2)}{5.717864} & if\ 0.14 \le \delta_y(A_1,A_2) \le 5.857864 \\ 0 & if\ \ \delta_y(A_1,A_2) \ge 5.857864 \end{cases}$$

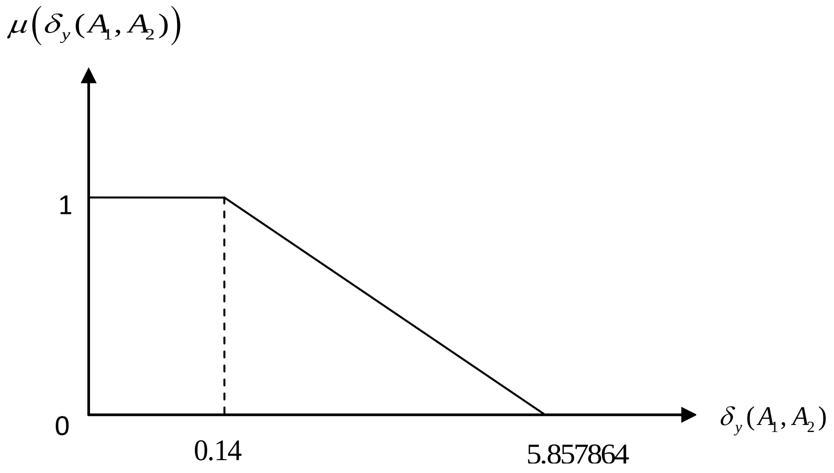

Figure 5. Membership and non-membership function for $\delta_y(A_1,A_2)$

Now Fuzzy decision making method used by weighted bounded sum operator (member of Yager family of triangular conorms) ,

$$Maximize\ \ F = W_1\mu\left(WT(A_1,A_2)\right)+W_2\mu\left(\delta_x(A_1,A_2)\right)+W_3\mu\left(\delta_y(A_1,A_2)\right) \tag{10}$$

subject to

$$0\le\mu\left(WT(A_1,A_2)\right)\le 1;\ 0\le\mu\left(\delta_x(A_1,A_2)\right)\le 1$$
$$0\le\mu\left(\delta_y(A_1,A_2)\right)\le 1$$
$$\mu\left(WT(A_1,A_2)\right)=\frac{15-WT(A_1,A_2)}{12}$$
$$\mu\left(\delta_x(A_1,A_2)\right)=\frac{20-\delta_x(A_1,A_2)}{16.5}$$
$$\mu\left(\delta_y(A_1,A_2)\right)=\frac{5.857864-\delta_y(A_1,A_2)}{5.717864}$$
$$\frac{20\left(2A_1+A_2\right)}{2A_1^2+2A_1A_2}\le 20;$$
$$\frac{20}{\sqrt{2}\left(A_1+A_2\right)}\le 10;$$
$$\frac{20\left(A_2\right)}{2A_1^2+2A_1A_2}\le 20;$$
$$0.1\le A_1,A_2\le 5,$$
$$W_1+W_2+W_3=1,$$

The solution obtained from Eq. (10) is given in table 2.

Table 2: Optimal results of MOSOP (9)

W_1	W_2	W_3	A_1^* $\times10^{-4}m^2$	A_2^* $\times10^{-4}m^2$	$WT^*(A_1,A_2)$ $\times10^2 KN$	$\delta_x^*(A_1,A_2)$ $\times10^{-7}m$	$\delta_y^*(A_1,A_2)$ $\times10^{-7}m$
1/3	1/3	1/3	2.661308	0.1029932	5.425610	7.375100	0.14
0.6	0.2	0.2	1.645225	0.1	3.390449	11.80812	0.3482759
0.2	0.6	0.2	4.542762	0.3	9.394060	4.262608	0.14
0.2	0.2	0.6	2.661309	0.1029933	5.425611	7.375099	0.14

In table 3, the value of R is quite small and hence the optimal results in table 2 are strong Pareto-optimal solution and can be accepted.

Table 3: Pareto optimal results of MOSOP (9)

R	A_1^{**} $\times 10^{-4} m^2$	A_2^{**} $\times 10^{-4} m^2$	$WT^{**}(A_1, A_2)$ $\times 10^2 KN$	$\delta_x^{**}(A_1, A_2)$ $\times 10^{-7} m$	$\delta_y^{**}(A_1, A_2)$ $\times 10^{-7} m$
0.5277774×10^{-6}	2.661308	0.1029933	5.425609	7.375100	0.14

IX. CONCLUSIONS

The present paper proposes a solution procedure for structural model. The results of this study may lead to the development of effective t-norm optimization method. This solution procedure may be used for solving other model of nonlinear programming problem in different field.

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Conflict of interests

The authors declare that there is no conflict of interests.

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